CRITICAL ULTRASONICS NEAR THE SUPERFLUID TRANSITION : FINITE SIZE EFFECTS

Saugata Bhattacharyya and J. K. Bhattacharjee
Department of Theoretical Physics
Indian Association for the Cultivation of Science
Jadavpur, Calcutta 700 032, India

Abstract

The suppression of order parameter fluctuations at the boundaries causes the ultrasonic attenuation near the superfluid transition to be lowered below the bulk value . We calculate explicitly the first deviation from the bulk value for temperatures above the lambda point . This deviation is significantly larger than for static quantities like the thermodynamic specific heat or other transport properties like the thermal conductivity . This makes ultrasonics a very effective probe for finite size effects .

PACS number(s): 64.60.Ht

Critical phenomena in confined geometry has been attracting a fair amount of attention of late because of the progress on the experimental front[$2-9$] which is making it possible to check the predictions of finite size effects (FSE). A fair amount of this experimental effort has gone in studying the specific heat near the superfluid transition . With the bulk specific heat quite well understood and the existence of a sharp phase transition (apart from gravity rounding , which too can be removed by doing experiments in space) established , efforts have been made to study the FSE . It is expected that the FSE will round out the transition and hence the divergence at $T = T_\lambda$ will be removed . The specific heat will be finite and the finite value will be a function of the confining length . We will keep in mind one of the favoured experimental geometries, where one takes two parallel plates separated by a distance L , much smaller than the linear dimensions of the plates . For $L \gg \xi$, the correlation length at a given temperature, usual thermodynamic result follows . It is when $L \leq \xi$, that FSE dominate . Finite size scaling suggests the existence of a scaling function , function of ξ/L- in terms of which the theory can be cast . The specific heat $C(t, L)$ in finite geometry has the form $C(t, L) \sim t^{-\alpha} g(t^{-\nu}/L) + Constant$, where $\xi \sim t^{-\nu}$ and $t = (T - T_\lambda)/T_\lambda$, T_λ being the transition temperature . The function **g** has been calculated by various authors [$10-12$] . In what follows we propose a method of checking for FSE by studying a related dynamic property . This is the study of ultrasonic attenuation (UA) near T_λ at high frequencies . In fact, it is our contention that UA is one of the best ways of checking for FSE since the single surface effect alone can produce effects greater than 10% . The critical fluctuations relax according to ξ^{-z}, where z is the dynamic scaling exponent . For frequencies ω such that $\omega \gg \Gamma_0$ (occurs if one is close to the critical point) , the attenuation is independent of the correlation length. For a finite size system , we will show below that in this limit , the attenuation is determined by ω and L alone . We provide explicit answers for frequencies ω which are much smaller than a cutoff frequency ω_0 (of the order of a few GHz) and for plate separation $L \geq (2\Gamma_0/\omega)^{\frac{1}{2}}$ for a given frequency ω . Our prediction for the attenuation per wavelength as a function of ω for the plate separation of $2110\mathring{A}$ is shown in Fig(1) . It should be possible to check the prediction experimentally . In fact , this should be the simplest way of checking for FSE since the effect is quite pronounced (about 18% at $L = 2110\mathring{A}$ and $\omega = 10Mhz$) for the available confining lengths as shown in Fig(1) . This occurs because the imaginary part of the specific heat

determines the UA and which is much smaller than the real part , but as we shall see below , **both are equally affected by the FSE** . Consequently, the relative effect is much larger for the imaginary part and this will show up in the UA .

The basis of our calculation is once more the Pippard Buckingham Fairbank (PBF) relation [$13-14$] which gives a successful account [$15-17$] of the critical ultrasonics in the situation where $L \gg \xi$. The PBF relation is obtained from general considerations of entropy clamping and yields for the sound velocity $u(T,\omega)$

$$u(T,\omega) = u_0(T_0) + u_1 C_0 / C_P(T,\omega) \tag{1}$$

where $u_0(T_0)$ is the sound speed at the transition point (T_0 is the bulk T_λ for the infinite system , but is a L-dependent temperature for the finite size system) , u_1 and C_0 are constants and $C_P(T,\omega)$ is the specific heat at finite frequency .

For the bulk case , $C_P(T,\omega=0)$ diverges at $T=T_\lambda$ and $C_P(T,\omega)$ is a homogeneous function of ω and ξ . If the characteristic relaxation rate is $\Gamma_0\xi^{-z}$ then the scaling form of C_P^{bulk} is

$$C_P^{bulk}(T,\omega) = \xi^{\alpha/\nu} f(\frac{\omega}{\Gamma_0\xi^{-z}}) \tag{2}$$

The exponent α is very close to zero for the superfluid transition in in ^4He and for many practical purposes , it is possible to write

$$C_P^{bulk}(T,\omega) = C[\ln(\Lambda\xi) + f(\frac{\omega}{\Gamma_0\xi^{-z}})] \tag{3}$$

The function $f(\omega/(\Gamma_0\xi^{-z}))$ reduces to a constant for $\omega = 0$ and tends to $-\ln(\omega/\Gamma_0)^{1/z}\xi$ for $\omega \gg \Gamma_0\xi^{-z}$. A one loop calculation of the scaling function $f(\Omega)$ where $\Omega = \frac{\omega}{\Gamma_0\xi^{-z}}$, was carried out and led to a successful scaling theory of the attenuation in the bulk ^4He near T_λ [$15-17$] .

We now need to discuss the effect of a confining geometry . At zero frequency , the specific heat is blunted due to the FSE and the usually divergent specific heat remains finite . The single loop calculation of the scaling function $g(\xi/L)$ discussed before gives a very reasonable account of the recent specific heat data by Mehta and Gasparini [2] . One of the most important feature of the scaling function is the low ξ/L limit (experimentally

most easily accessible) is the first departure from the thermodynamic limit - the magnitude of this departure ΔC has to be proportional to the surface (A) to volume (V) ratio and hence from purely dimensional arguments , the correction can be written as

$$\Delta C = C(\xi, L) - C_\infty(\xi) = -aCA\frac{\xi}{V} \tag{4}$$

where a is a number of $O(1)$, that can be obtained from the function $g(\xi/L)$, and C is the dimensional constant defined in eqn(3) . The value of a as inferred from Scmolke et al. [11] is 1.4. The agreement of this departure with the measured departure of Mehta and Gasparini is impressive .

In our present concern we need the three variable function $C(\xi, L, \omega)$, whose two limits $C(\xi, \omega)$ and $C(\xi, L)$ are already well known . We will characterize $C(\xi, L, \omega)$ by its first departure from the infinite volume limit $C(\xi, \omega)$ and write the generalization of eqn(4) as

$$\Delta C(\xi, L, \omega) = C(\xi, L, \omega) - C(\xi, \omega) = -a(\xi, \omega)C(\xi)A/V \tag{5}$$

where $a(\xi, \omega)$ is a scaling function , whose zero frequency limit is $a\xi$ (see Eqn(4)) and whose general form will be presented below . As soon as we start discussing the scaling function for $C(\xi, L, \omega)$ we need to worry about what sets the scale for ω . As we have discussed above , this has to be the rate of decay of fluctuations $\Gamma(\xi)$. In the finite geometry that we are discussing now , the scale for decay of fluctuations will depend on L as well . In discussing the correction depicted in Eqn(5) , it is obvious that this fine point need not be discussed as this correction is already $O(\frac{1}{L})$. For He (superfluid transition) , there is in someways an additional simplifying feature . For the order parameter decay rate the non-linear effect of fluctuations becomes significant , only very close to the critical point and for all practical purposes , the relaxation rate can be taken to be at its non critical background value .

The complex order parameter field $\psi_i(x) i = 1, 2$ will be governed by the Langevin equation

$$\dot{\psi}_i = -\Gamma_0 \frac{\delta F}{\delta \psi_i} + N_i \tag{6}$$

where

$$F = \int d^D x [\frac{m^2}{2}\psi^2 + \frac{1}{2}(\nabla\psi)^2 + \frac{\lambda}{4}(\psi^2)^2] \tag{7}$$

and N is a Gaussian white noise . For reasons stated above we choose to drop the reversible term (the Josephson equation for the phase of the order parameter) . The parameter m^2 is proportional to $T - T_\lambda$, where T_λ is the bulk transition temperature . The system is confined in one of the D directions . We call that the z-direction . It is convenient to work with the fourier transform in $D-1$ directions and the fourier series (Dirichlet boundary conditions at $z=0$ and $z=L$ suppressing the fluctuations) in the z direction . The expansion of the time-dependent order parameter field is

$$\psi_i(\vec{r},t) = \sum_n \psi_i(n,K,t) \exp^{i\vec{K}.\vec{R}} \sin(\frac{n\pi z}{L}) \tag{8}$$

The equation of motion for $\psi_i(n,K,t)$ is

$$\dot{\psi}_i(n,K,t) = -\Gamma_0(m^2 + K^2 + \frac{n^2\pi^2}{L^2})\psi_i(n,K,t) + N_i + O(\psi^3) \tag{9}$$

In what follows , we will assume that all static correlations have been accounted for and $m^2 = \xi^{-2}$. The specific heat is obtained as the response function corresponding to the time dependent correlation function

$$D(\xi,L,t_{12}) = \int\int\int dz_1 dz_2 d^D R_{12} < \psi^2(\vec{R_1},z_1,t_1)\psi^2(\vec{R_2},z_2,t_2) > \tag{10}$$

with $D(\xi,L,\omega) = 2\frac{ImC(\xi,L,\omega)}{\omega}$ according to fluctuation dissipation theorem , straightforward algebra leads to (two term accuracy , $L \to \infty$ limit and the first correction)

$$\begin{aligned} C(\xi,L,\omega) &= \int \frac{d^D p}{(2\pi)^D} \frac{1}{(p^2+m^2)} \frac{1}{(-\frac{i\omega}{2\Gamma_0} + p^2 + m^2)} \\ &\quad -\frac{1}{2L}\int \frac{d^{D-1}p}{(2\pi)^{D-1}} \frac{1}{(p^2+m^2)} \frac{1}{(-\frac{i\omega}{2\Gamma_0} + p^2 + m^2)} \end{aligned} \tag{11}$$

We work to logarithmic accuracy and hence evaluate the integrals at $D=4$ (proper exponentiation can be undertaken by working to two loop order , the details of which will be published elsewhere) to get the functions $f(\Omega)$ and $a(\Omega)$ introduced in Eqs(3) and (5) . Note that since we are taking the logarithmic divergence for the bulk specific heat , the $C(\xi)$ in Eqns(4) and (5) reduces the constant C of Eqn(3) . The function $f(\Omega)$ and $a(\Omega)$ are

$$f(\Omega) = \frac{1}{2}(\frac{1}{-i\Omega} - 1)\ln(1 - i\Omega) \tag{12}$$

$$a(\Omega) = \frac{\pi}{2}\frac{1}{-i\Omega}[\sqrt{1-i\Omega} - 1] \tag{13}$$

leading to

$$\begin{aligned} C(\xi, L, \omega) &= C_0\{\ln\frac{\Lambda}{m} - \frac{1}{4}\ln(1+\Omega^2) - \frac{1}{2\Omega}\tan^{-1}(\Omega) + i[\frac{1}{2}\tan^{-1}(\Omega) \\ &\quad -\frac{1}{4\Omega}\ln(1+\Omega^2)] - \frac{\pi}{mL\Omega}(1+\Omega^2)^{\frac{1}{4}}\sin(\frac{\tan^{-1}\Omega}{2}) - \frac{i\pi}{mL\Omega} \\ &\quad [(1+\Omega^2)^{\frac{1}{4}}\cos(\frac{\tan^{-1}\Omega}{2}) - 1]\} \\ &= C_R + iC_I \end{aligned} \tag{14}$$

where C_R and C_I are the real and imaginary parts of the specific heat. Considering the zero frequency limit we see that $C_R = C_0[\ln\Lambda/m - \pi/2mL]$ leading to $a = \pi/2$ in Eq. (4) which is to be compared with $a \simeq 1.4$ obtained in [11].

We now return to Eqn(1) , to find the attenuation and dispersion . The **attenuation per wavelength** is $\frac{\alpha\lambda}{2\pi} = \frac{u_1 C_0 C_I}{u_0(C_R^2+C_I^2)}$ which leads to the frequency attenuation ($\omega \gg 2\Gamma_0 m^2$) as

$$\frac{\alpha\lambda}{2\pi} = \frac{\pi u_1}{u_0}\frac{[1-2\sqrt{2}(\frac{2\Gamma_0}{\omega L^2})^{\frac{1}{2}}]}{[\ln(\frac{\omega_0}{\omega}) - \sqrt{2}\pi(\frac{2\Gamma_0}{\omega L^2})^{\frac{1}{2}}]^2 + \frac{\pi^2}{4}[1-2\sqrt{2}(\frac{2\Gamma_0}{\omega L^2})^{\frac{1}{2}}]^2} \tag{15}$$

This is the **saturation attenuation** per wavelength , which does not change as the temperature is lowered further , where $\omega_0/2\pi = 30GHz, \Gamma_0 = 1.2 \times 10^{-4} cm^2 sec^{-1}, u_1/u_0 = 8/3 \times 10^{-2}$.

For the plate separation of $2110\mathring{A}$ of mehta and Gasparini , the reduction in the attenuation due to the quenching of fluctuations is about 18% at $10MHz$ and increases to 45% at $2.5MHz$. This is a large effect compared to the 4% surface effects that show up in the static measurements . For the corresponding measurement of thermal conductivity near the superfluid transition Kahn and Ahlers [9] found that the deviation from the bulk is about 7% when the correlation length ξ equals the confining length L (in their case the radius of the pore) . The surface effect for the ultrasonic measurement can easily amount to 30% which makes this an attractive system for a confrontation between theory and experiment . The FSE on the dispersion can be obtained from the real part of Eqn(1).

We note that the above is a one loop calculation in the critical region . The lack of crossover to the background in our treatment of the specific heat implies that we can consider frequency ω which are much smaller than the cut-off frequency ω_0 . This is a restriction on the validity of the dashed curve shown in Fig(1) . The solid curve in addition is restricted to confining lengths which are not too small i.e. $L \geq (2\Gamma_0/\omega)^{\frac{1}{2}}$ and in this regime the accuracy of the calculation is restricted by the loop order . This is not too severe a restriction as an accuracy of $O(\epsilon)$ which our calculation entails,becomes an accuracy of $O(\frac{\alpha}{\nu})$ when the combinatorial factors are included . Thus, in the above mentioned ranges of the parameters ω and L , the dashed curve in Fig(1) should be an accurate prediction . It should be noted that contrary to the static specific heat or the thermal conductivity , the sound properties can only be probed in a real experiment . The final issue , then , is whether the effect can be observed in real a experiment . The critical ultrasonics near the superfluid transition has been studied more than two decades ago . The most accurate data of that period lies in the $0.5MHz - 5MHz$ range . In this region the scatter in the data is about 15%. This is somewhat better than the borderline for detecting the suppression reported here . Considering the fact that developments in the experimental field would enable more accurate measurements at the present time , we believe this effect should be experimentally accessible .

The other sensitive part of an ultrasonic measurement is the low frequency end ($\omega \ll 2\Gamma_0 m^2$) , where for the bulk substance the attenuation per wavelength is proportional to $C_R^2\Omega/4$. The relative correction for the FSE is $1 - \frac{\pi}{2mL}$, once again a larger effect than can be obtained in statics. For an easily realizable situation of $ml \sim 8$ this gives a 20% reduction in the attenuation . The whole course of the attenuation function with its dependence on ω and L is straightforward to obtain and will be exhibited elsewhere . Here we have reported the salient feature , which carry the most experimentally accessible signatures . We hope this will stimulate experimental activity in the field.

Acknowledgments

One of the authors (SB) would like to thank the C.S.I.R, India, for pro-

viding partial financial support and Dr.Manabesh Bhattacharya for his help and encouragement. We thank the referee for pointing out various improvements.

References

[1] For a review see, V.Dohm, Physica Scripta

[2] S. Mehta and F. M. Gasparini, Proceedings of LT-21 Czech. J.Phys **46** Suppl S1 173 (1996) ,Phys. Rev. Lett **78** 2596 (1997)

[3] J. P. Chen and F. M. Gasparini, Phys. Rev. Lett **40** 331 (1970)

[4] F. M. Gasparini , G. Agnolet and J. D. Reppy, Phys. Rev. **B29** 138 (1984)

[5] I. Rhee , F. M. Gasparini and D. J. Bishop, Phys. Rev. Lett. **63** 410 (1989)

[6] J. A. Nissev, T. C. P. Chui and J. A. Lipa, J.Low Temp. Phys **92** 393 (1993)

[7] M. Coleman and J. A. Lipa, Proceedings of LT-21, Czech. J. Phys. **46** S1 183 (1996)

[8] G. Ahlers and R. V. Duncan, Phys. Rev. Lett.**61** 846 (1988)

[9] A. M. Kahn and G. Ahlers, Phys. Rev. Lett **73** 944 (1995)

[10] W. Huhn and V. Dohm , Phys. Rev. Lett **61** 1368 (1988)

[11] R. Schmolke, A. Wacker, V. Dohm and D. Frank , Physica **B165 166** 575 (1990)

[12] P. Sutter and V. Dohm , Physica**B** (1993)

[13] A. B. Pippard, Phil. Mag. **1** 473 (1956)

[14] M. J. Buckingham and W. M. Fairbank , Prog. in Low Temp. Phys. ,ed. by C. J. Gorter (North-Holland , Amsterdam) **III** 80 (1961)

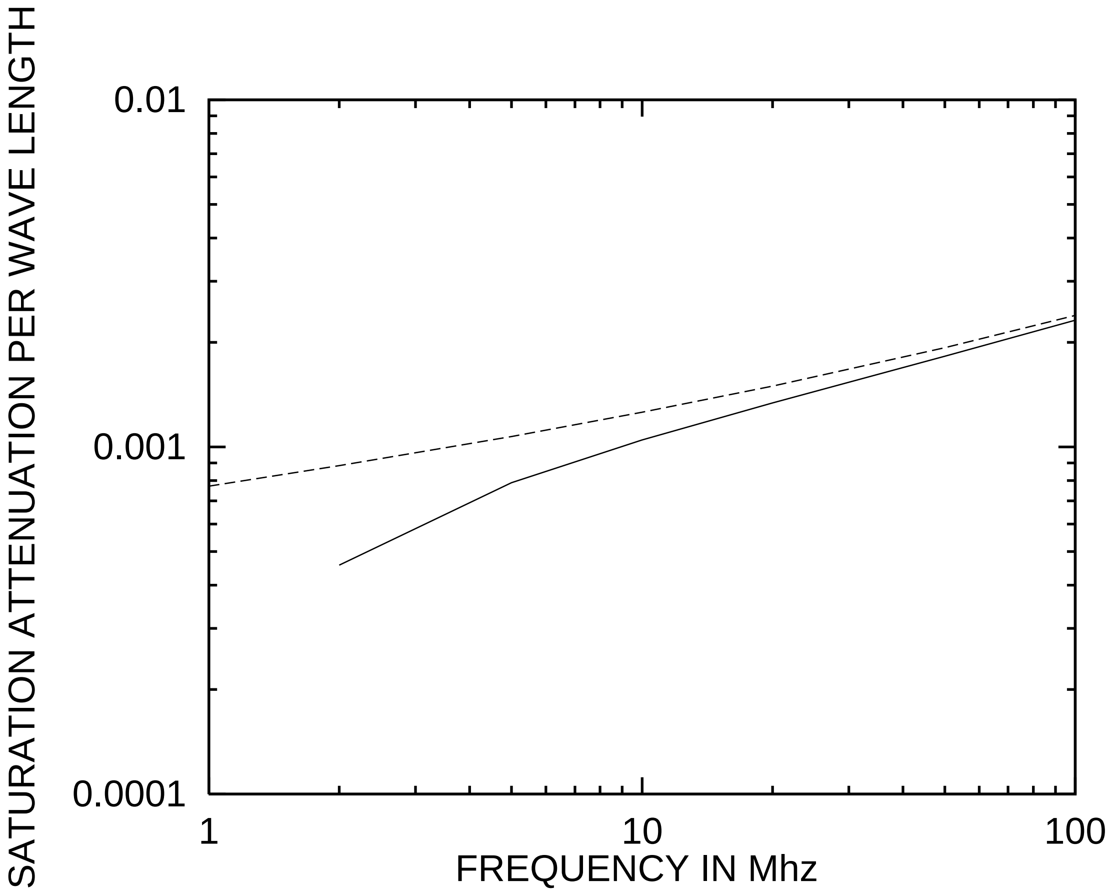
[15] R. A. Ferrell and J. K. Bhattacharjee , Phys. Rev. Lett **44** 403 (1982)

[16] R. A. Ferrell and J. K. Bhattacharjee , Phys. Rev. **B23** 2484 (1981)

[17] J. Pankert and V. Dohm , Phys. Rev.**B40** 10856 (1989)

Figure caption

Fig.1. Saturation attenuation is plotted against frequency.The dashed curve shows the bulk ($L \to \infty$) result whereas the solid curve shows the surface effect.


SATURATION ATTENUATION PER WAVE LENGTH
0.01
0.001
0.0001
1
10
100
FREQUENCY IN Mhz